

# The origin of mass and a counterargument to the special theory of relativity

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Reasoning in accordance with the special theory of relativity produces some strange consequences. For example, we find contradictions in the cases mentioned next. Case 1) Assume that two electrons run parallel in a vacuum at a fixed distance with the same speed. We may consider this as a case of parallel current in which case, in addition to electrostatic repulsion, magnetic attraction proportional to speed will also act. At the same time we may think of the system as a static coordinate system, in which case the two electrons are at rest and only electrostatic repulsion will act between the two electrons. We thus have a contradiction in that the same matter exhibits different forces according to our view of static systems. Case 2) Let us assume that in a certain system an electron at rest comes into Compton collision with an electromagnetic wave and gains energy and velocity. If we now consider this speed system as a static system, then in this system the electron is at rest. If this electron again comes into collision with an electromagnetic wave advancing in the reverse direction to the first collision and returns to the original static state, then the electromagnetic energy gained from the two collisions has disappeared and been lost. This is a further contradiction.

The premise of the special theory of relativity is that the speed of electromagnetic waves is constant in any speed system. This leads to the consequence that, in different speed systems, in order to satisfy the Lorentz transformation, time progresses more slowly and distance is reduced. Since the Lorentz transformation is used to explain many phenomena of physics, it cannot be lightly rejected. To solve these problems in this situation, we need to be able to account for phenomena currently explained by the Lorentz transformation by another method. In this paper we attempted to explain the following three topics: the Fizeau empirical formula (velocity of light in a stream), the Michelson-Moley experiment, the Compton effect and explanation was completed without using the Lorentz transformation. Moreover it was expected that the relation between velocity and energy of the recoil electron in the Compton effect could be conversely reasoned from the result of the Compton effect. However, explanation of the velocity of an electron was not successful without taking into consideration a new concept which treats an electron similarly to a wave motion. An electron needs to be considered as one form of a wave, and its velocity calculated by the Doppler effect. A relational equation for the equivalence of energy and mass and the increase of mass according to speed is derived from this hypothesis. Furthermore, it was found with the newly introduced concept that “energy product” is conserved, and an equation of velocity which indicates the origin of mass is obtained.

## Explanation of the Fizeau empirical formula

This formula was first explained by the composition of velocity within the special theory of relativity, and the phenomenon cannot be explained through the ordinal composition of velocity. Using a refractive index  $n$  inside a substance, the velocity of light is shown by  $c/n$ . Here we shall consider that the velocity of light inside a substance is identical with that in a vacuum, but that owing to refraction it travels more distance than if going straight. This results in  $c/n$ . For example, light appears to travel distance  $L$  but actually travels distance  $nL$  inside a substance as shown in Figure 1a.

Here  $t$  is the time to travel from a particle to a particle and  $V$  the speed of light. This can be described as  $t = nL/c$ , therefore

$$V = \frac{L}{t} = \frac{L}{\frac{nL}{c}} = \frac{c}{n}$$

This interpretation of refraction is that light is refracted by particles and angle  $\theta$  becomes the average refracting angle. Consequently we may consider that the velocity of light becomes  $c \cos \theta$ .

From this perspective we attempt to calculate the change of the velocity of light in the case where particles move in the direction in which light advances. As shown in Figure 1b, where a particle moves with speed  $v$  in the same direction as light, a particle which was situated at point A in the state of rest moves to point A'. The new variable,  $\theta = \angle ABC$  and  $\phi = \angle A'BC$ . The constant  $c$  represents the velocity of light in a vacuum,  $n$  is the refractive index,  $t_1$  is the time to advance from point B to point A, and  $t_2$  the time to advance from point B to point A'.

Since distance  $BA = ct_1$ ,  $BC = BC + CC'$ , and  $AC = BA \cos \theta = A'C = BA' \cos \phi$ , we obtain

$$ct_1 \cos \theta + vt_2 = ct_2 \cos \phi \quad \dots \quad (1)$$

$$ct_1 \sin \theta = ct_2 \sin \phi \quad \dots \quad (2)$$

Equations (1) and (2) reduce to

$$\cos \theta = \frac{t_2}{t_1} \cos \phi - \frac{v t_2}{c t_1}, \quad \sin \theta = \frac{t_2}{t_1} \sin \phi$$

Substituting these equations for equation  $\sin^2 \theta + \cos^2 \theta = 1$

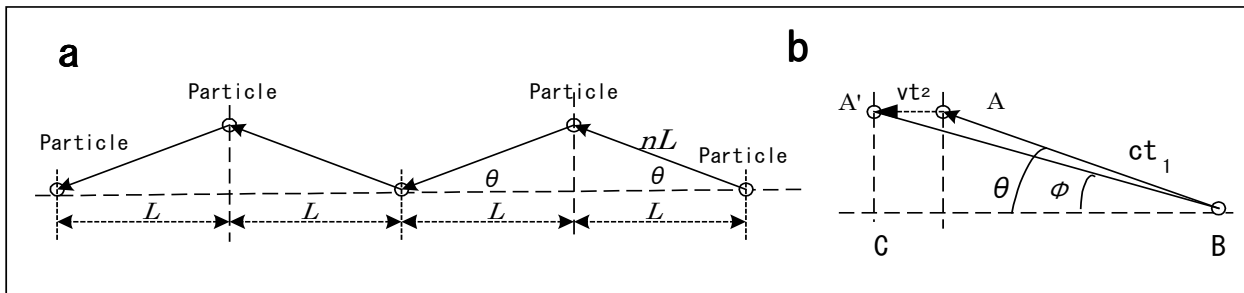
$$\therefore \left(\frac{t_2}{t_1} \sin \phi\right)^2 + \left(\frac{t_2}{t_1} \cos \phi - \frac{v t_2}{c t_1}\right)^2 = 1$$

$$\therefore \sin^2 \phi + \left(\cos^2 \phi - 2 \frac{v}{c} \cos \phi + \frac{v^2}{c^2}\right) = \left(\frac{t_1}{t_2}\right)^2 \quad \dots \quad (3)$$

Substituting  $\cos \theta = 1/n$  for equation (1)

$$ct_1 \frac{1}{n} + vt_2 = ct_2 \cos \phi, \quad \therefore \frac{t_1}{t_2} = n \left(\cos \phi - \frac{v}{c}\right) \quad \dots \quad (4)$$

Substituting (4) for (3)



**Figure 1. a, The relation of the speed of light and refractive index.** Light appears to travel distance ( $L$ ) but actually travels distance ( $nL$ ). Here  $t$  is the time to travel from a particle to a particle and  $V$  the speed of light. This can be described as  $t = nL/c$ . **b, Change of the refractive index according to movement of substance.** Particles move with speed  $v$  in the same direction as light. A particle substituted at point A in the state of rest moves to point A'. Since  $\theta = \angle ABC$  and  $\phi = \angle A'BC$ , then refractive angle  $\theta$  changes to  $\phi$ . Consequently the velocity of light becomes  $c \cos \phi$ .  $t_1$  is the time to travel to point A' from point B, and  $t_2$  the time to travel to point A from point B.  $n$  is the refractive index of a substance, and  $c$  the speed of light

$$\begin{aligned} \therefore 1 - 2\frac{v}{c}\cos\phi + \frac{v^2}{c^2} &= n^2\left(\cos^2\phi - 2\frac{v}{c}\cos\phi + \frac{v^2}{c^2}\right) \\ \therefore \cos\phi - 2\frac{v}{c}\cos\phi\left(1 - \frac{1}{n^2}\right) + \frac{v^2}{c^2}\left(1 - \frac{1}{n^2}\right) - \frac{1}{n^2} &= 0 \\ \therefore \cos\phi &= \frac{v}{c}\left(1 - \frac{1}{n^2}\right) \pm \sqrt{\frac{1}{n^2} - \frac{v^2}{c^2}\left(1 - \frac{1}{n^2}\right) + \frac{v^2}{c^2}\left(1 - \frac{1}{n^2}\right)^2} \quad \dots \quad (5) \end{aligned}$$

Since  $v \ll c$ , the expression  $(v/c)^2$  approximates to 0, and equation (5) reduces to

$$\cos\phi = \frac{v}{c}\left(1 - \frac{1}{n^2}\right) \pm \frac{1}{n}$$

Consequently as the composition of velocity  $V'$ , we obtain

$$V' = c \cos\phi = v\left(1 - \frac{1}{n^2}\right) \pm \frac{c}{n} \quad \dots \quad (6)$$

This is the Fizeau empirical formula itself.

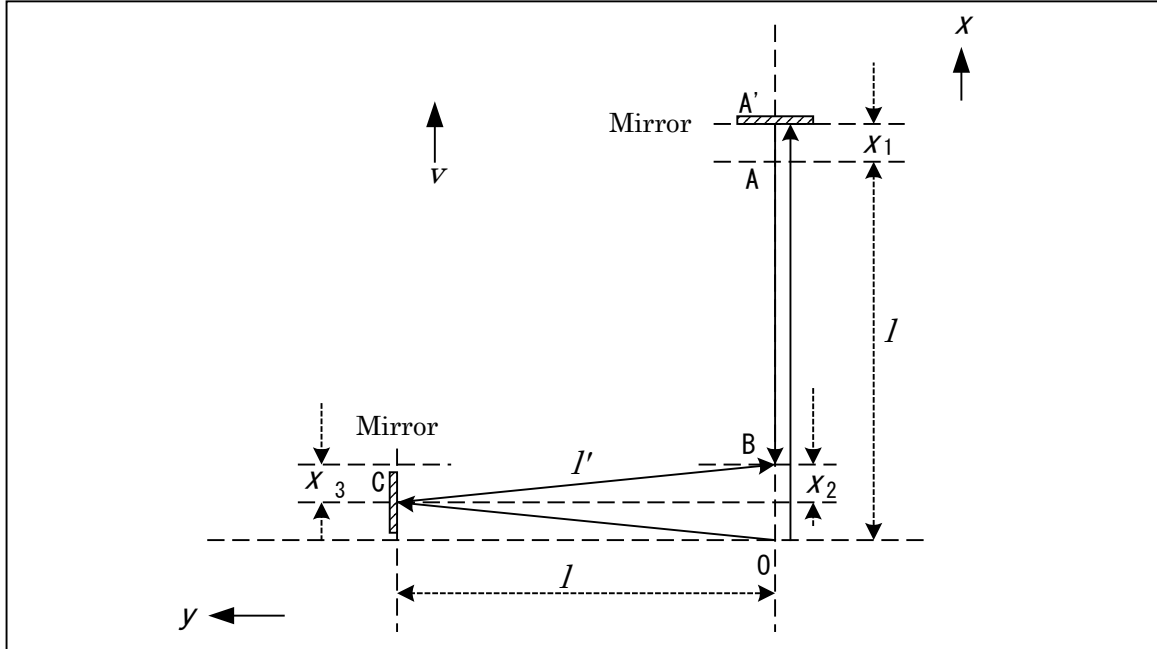
### Explanation of the Michelson-Moley experiment

This explanation will be made with the same reasoning as for the Fizeau empirical formula. As shown in Figure 2, the experimental device is moving with speed  $v$

#### Calculation of the time $T_1$ of the round trip of light along the x axis (O->A'->B).

Here we can calculate  $t_1$  and  $t_2$  by the Fizeau empirical formula. We obtain

$$t_1 = OA' / \left\{ \frac{c}{n} + v\left(1 - \frac{1}{n^2}\right) \right\}, \quad t_2 = A'B / \left\{ \frac{c}{n} - v\left(1 - \frac{1}{n^2}\right) \right\}$$



**Figure 2. Set-up conditions for the Michelson-Moley experiment.** The whole device is moving with speed  $v$ .  $t_1$  is the time taken for the mirror situated at point A to move to point A' when light is reflected along the x axis.  $t_2$  is the time taken for light to return from point A' to point B.  $x_1$  is the distance which the device moves in time  $t_1$ , and  $x_2$  the distance which the device moves in time  $t_2$ .  $l$  is the distance between the light source and the reflecting mirror, and C is the reflection point along the y axis.  $2x_2 = OB$ .

$$t_1 = \frac{l + x_1}{\frac{c}{n} + v(1 - \frac{1}{n^2})} = \frac{l + vt_1}{\frac{c}{n} + v(1 - \frac{1}{n^2})},$$

$$\therefore t_1 \frac{c}{n} + t_1 v - t_1 \frac{v}{n^2} = l + vt_1 \quad \therefore t_1 = \frac{l}{\frac{c}{n} - \frac{v}{n^2}} \quad \dots (1)$$

$$t_2 = \frac{l - x_2}{\frac{c}{n} - v(1 - \frac{1}{n^2})} = \frac{l - vt_2}{\frac{c}{n} - v(1 - \frac{1}{n^2})},$$

$$\therefore t_2 \frac{c}{n} - t_2 v + t_2 \frac{v}{n^2} = l + vt_2 \quad \therefore t_2 = \frac{l}{\frac{c}{n} + \frac{v}{n^2}} \quad \dots (2)$$

therefore

$$\therefore T_1 = t_1 + t_2 = \frac{l}{\frac{c}{n} - \frac{v}{n^2}} + \frac{l}{\frac{c}{n} + \frac{v}{n^2}} = \frac{l \cdot 2 \frac{c}{n}}{(\frac{c}{n})^2 - (\frac{v}{n^2})^2} = \frac{2l}{\frac{c}{n} \left\{ 1 - (\frac{v}{cn})^2 \right\}} \quad \dots (3)$$

**Calculating the distance,  $l'$ , of  $y$  axis orientations (O → C · C → B)**

$T_2$  is the time taken for light to travel along the  $y$  axis orientations (O → C → B), and  $x_3$  is the distance (OB/2).

Distance  $x_3$  is

$$x_3 = \frac{1}{2}(x_1 + x_2) = \frac{v}{2}(t_1 + t_2)$$

Substituting equation (3) in this equation

$$x_3 = \frac{vl}{\frac{c}{n} \left\{ 1 - (\frac{v}{cn})^2 \right\}} \quad \dots (4)$$

Then distance  $l'$  is given as below

$$l' = \sqrt{x_3^2 + l^2}$$

$$l' = l \sqrt{\frac{v^2}{\left( \frac{c}{n} \left\{ 1 - (\frac{v}{cn})^2 \right\} \right)^2} + 1}$$

$$= \frac{l}{\frac{c}{n} \left\{ 1 - (\frac{v}{cn})^2 \right\}} \sqrt{v^2 + \left( \frac{c}{n} \right)^2 \left\{ 1 - (\frac{v}{cn})^2 \right\}^2}$$

$$= \frac{l \left( \frac{c}{n} \right)}{\frac{c}{n} \left\{ 1 - (\frac{v}{cn})^2 \right\}} \sqrt{\frac{n^2}{c^2} v^2 + 1 - 2 \left( \frac{v}{cn} \right)^2 + \left( \frac{v}{cn} \right)^4}$$

Since  $v \ll c$ , value  $n$  approximates to 1, and we can describe

$$l' = \frac{l}{\left\{1 - \left(\frac{v}{cn}\right)^2\right\}} \sqrt{1 - \left(\frac{v}{cn}\right)^2} = \frac{l}{\sqrt{1 - \left(\frac{v}{cn}\right)^2}} \quad \dots \quad (5)$$

**The speed of light where the moving system of the experimental device is at right-angles to the direction of light**

In the case of the Fizeau empirical formula, the motion of the laboratory system is parallel to the direction of light, but in this case the system of the experiment is at right-angles to the direction of light. Here we treat refraction and the speed of light in the same way as in the explanation of the Fizeau empirical formula. As shown in Figure 3, the experimental device moves at a fixed speed  $v$  in the direction of the  $x$  axis and light travels to the  $y$  axis, and point A is where the light first refracts in the case of a static system. Since this system moves at speed  $v$ , point A moves to point A',  $\theta$  is angle  $\angle AOC$ , and  $\phi$  is angle  $\angle A'OC$ . At the same time the next refraction point moves from point B to point B', so that distance BB' must be equal to distance AA'. As light advances from point A' towards B', point B' moves to point B'', so that  $\phi'$  is angle  $\angle B''A'D$ , and now  $\phi' < \theta$ . Consequently the speed of light which we need to calculate must be  $c (\cos \phi + \cos \phi') / 2$ . Let us calculate these values.  $t_1$  is the time taken to move distance OA,  $t_2$  the time taken to move distance OA', and  $n$  is the refractive index in air.

From  $CA' = CA + AA'$

$$ct_1 \sin \theta + vt_2 = ct_2 \sin \phi \quad \dots \quad (6)$$

From  $OC = OA \cos \theta = OA' \cos \phi$

$$ct_1 \cos \theta = ct_2 \cos \phi \quad \dots \quad (7)$$

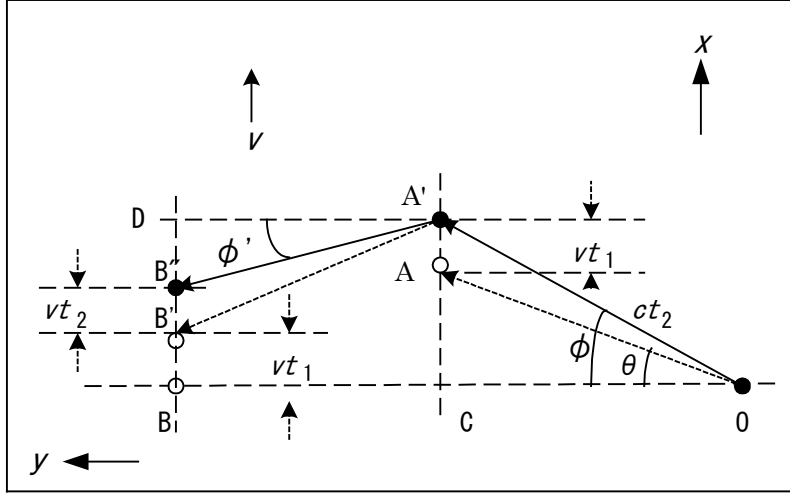
$$\therefore \sin \theta = \frac{t_2}{t_1} \left( \sin \phi - \frac{v}{c} \right) \quad \cos \theta = \frac{t_2}{t_1} \cos \phi$$

Substituting the above equations for  $\sin^2 \theta + \cos^2 \theta = 1$ , we obtain

$$\begin{aligned} \left(\frac{t_2}{t_1}\right) \left\{ \sin \phi - \left(\frac{v}{c}\right) \right\}^2 + \left(\frac{t_2}{t_1}\right) \cos^2 \phi &= 1 \\ \therefore 1 - 2\frac{v}{c} \sin \phi + \left(\frac{v}{c}\right)^2 &= \left(\frac{t_1}{t_2}\right)^2 \quad \dots \quad (8) \end{aligned}$$

$\cos \theta = 1/n$  and equation (6) modifies into

$$\frac{t_1}{t_2} = \frac{\sin \theta - \frac{v}{c}}{\sin \theta} \quad \therefore \left(\frac{t_1}{t_2}\right)^2 = \frac{\sin^2 \phi - 2\frac{v}{c} \sin \phi + \left(\frac{v}{c}\right)^2}{1 - \frac{1}{n^2}} \quad \dots \quad (9)$$



**Figure 3. The speed of light where the moving system of the experimental device is at right-angles to the direction of light.** The whole experimental device moves at fixed speed  $v$  in the direction of the  $x$  axis and light travels along the  $y$  axis. Point  $A$  is where the light first refracts in the case of a static system. Since this system moves at speed  $v$ , point  $A$  moves to point  $A'$ ,  $\theta$  is angle  $\angle AOC$ , and  $\phi$  is angle  $\angle A'OC$ . At the same time the next refraction point moves from point  $B$  to point  $B'$  so that distance  $BB'$  must be equal to distance  $AA'$ . As light advances from point  $A'$  towards  $B'$ , point  $B'$  moves to point  $B''$ , so that  $\phi'$  is angle  $\angle B''A'D$ , and now  $\phi' < \theta$ . Here  $A'D$  and  $OB$  are set as parallel.

Combining equations (8) and (9)

$$1 - 2\frac{v}{c}\sin\phi + \left(\frac{v}{c}\right)^2 = \frac{\sin^2\phi - 2\frac{v}{c}\sin\phi + \left(\frac{v}{c}\right)^2}{1 - \frac{1}{n^2}}$$

$$\therefore 1 - \frac{1}{n^2} - 2\frac{v}{c}\sin\phi\left(1 - \frac{1}{n^2}\right) + \left(\frac{v}{c}\right)^2\left(1 - \frac{1}{n^2}\right) = \sin^2\phi - 2\frac{v}{c}\sin\phi + \left(\frac{v}{c}\right)^2$$

$$\therefore \sin^2\phi - 2\frac{v}{cn^2}\sin\phi + \left\{\frac{1}{n^2} + \left(\frac{v}{cn}\right)^2\right\} - 1 = 0$$

$$\therefore \sin\phi = \frac{v}{cn^2} \pm \sqrt{1 - \frac{1}{n^2} - \left(\frac{v}{cn}\right)^2 + \left(\frac{v}{cn^2}\right)^2}$$

$$\therefore \sin\phi = \frac{v}{cn^2} \pm \sqrt{1 - \frac{1}{n^2}} \cdot \sqrt{1 - \left(\frac{v}{cn}\right)^2} \quad \dots \quad (10)$$

Next, to determine  $\cos\phi$

$$\cos^2\phi = 1 - \sin^2\phi$$

$$= 1 - \left\{\frac{v^2}{c^2n^4} \pm \sqrt{1 - \frac{1}{n^2}} \cdot \sqrt{1 - \left(\frac{v}{cn}\right)^2}\right\}^2$$

$$= 1 - \left\{\frac{v^2}{c^2n^4} \pm 2\frac{v}{c}\sqrt{1 - \frac{1}{n^2}} \cdot \sqrt{1 - \left(\frac{v}{cn}\right)^2} + 1 - \frac{1}{n^2} - \left(\frac{v}{c}\right)^2 + \left(\frac{v}{cn^2}\right)^2\right\}$$

$$= \frac{1}{n^2} \left\{1 - \left(\frac{v}{cn}\right)^2 \pm 2\frac{v}{c}\sqrt{1 - \frac{1}{n^2}} \cdot \sqrt{1 - \left(\frac{v}{cn}\right)^2} + \left(\frac{v}{c}\right)^2\left(1 - \frac{1}{n^2}\right)\right\}$$

$$\therefore \cos \phi = \frac{1}{n} \left\{ \sqrt{1 - \left(\frac{v}{cn}\right)^2} \pm \frac{v}{c} \sqrt{1 - \frac{1}{n^2}} \right\} \quad \dots \quad (11)$$

Therefore, returning to Figure 4, we can state

$$\therefore \cos \phi = \frac{1}{n} \left\{ \sqrt{1 - \left(\frac{v}{cn}\right)^2} + \frac{v}{c} \sqrt{1 - \frac{1}{n^2}} \right\}, \quad \cos \phi' = \frac{1}{n} \left\{ \sqrt{1 - \left(\frac{v}{cn}\right)^2} - \frac{v}{c} \sqrt{1 - \frac{1}{n^2}} \right\}$$

$V_2$ , the velocity of light to be determined here, is

$$V_2 = \frac{1}{2} c (\cos \phi + \cos \phi') = \frac{c}{n} \sqrt{1 - \left(\frac{v}{cn}\right)^2}$$

### Calculating the time $T_2$ of the round trip along the y axis (O → C → B in Figure 3)

$T_2$ , the time of the round trip along the y axis, is  $2 \cdot OC / V_2$

$$T_2 = \frac{2l'}{V_2} = \frac{2l}{\sqrt{1 - \left(\frac{v}{cn}\right)^2}} \cdot \frac{1}{\frac{c}{n}} \cdot \frac{1}{\sqrt{1 - \left(\frac{v}{cn}\right)^2}} = \frac{2l}{\frac{c}{n} \left\{ 1 - \left(\frac{v}{cn}\right)^2 \right\}} \quad \dots \quad (12)$$

$T_1$  in equation (3) is equal to  $T_2$  in equation (12), in addition, equation (3) and (12) are established by any n, consequently the round trip time in the direction of the x axis and in the direction of the y axis are equal, therefore movement of the ether may not be detected by this experiment, so nothing is answered about the existence of the ether.

An experiment of Michelson-Moley is the experiment that led to the special theory of relativity birth, but because the negation of the existence of the ether as the medium of light or the electromagnetic wave is not possible, the ground of the special theory of relativity would be lost.

### A reconsideration of the concept of energy to explain the Compton Effect

$(mv^2/2)$  is defined as kinetic energy when a particle with mass  $m$  moves at speed  $v$ . In other motion systems, it produces a different energy value with different speed values. This appears as an inconsistency from the viewpoint of the pure conservation of energy. Under the law of conservation of energy, calculation for arbitrary speed systems produces the right answer without inconsistency. Therefore it appears preferable to consider energy as an expedient tool for calculation rather than as a concept. In the case of the collision of a perfect elastic ball, however, calculation by the laws of conservation of energy and conservation of momentum produces a quadratic equation with regard to speed and two solutions result. One solution is eliminated according to the actual conditions, and the other one is regarded as correct.

From the viewpoint of logic, we should expect that there will be a method of calculation by means of linear equations for speed which will produce a single solution. In this way we attempt to calculate the case of two perfect elastic balls colliding on a straight line and flying away along the same straight line in the reverse direction. In Figure 4, the mass of each ball is  $m_1$  and  $m_2$ , and their speeds before collision are  $v_1$  and  $v_2$ . At the moment of collision, the impulse in the reverse direction works mutually until each ball reaches the speed of a barycentric coordinate system.  $v_3$  is the speed of a barycentric coordinate system, therefore we obtain

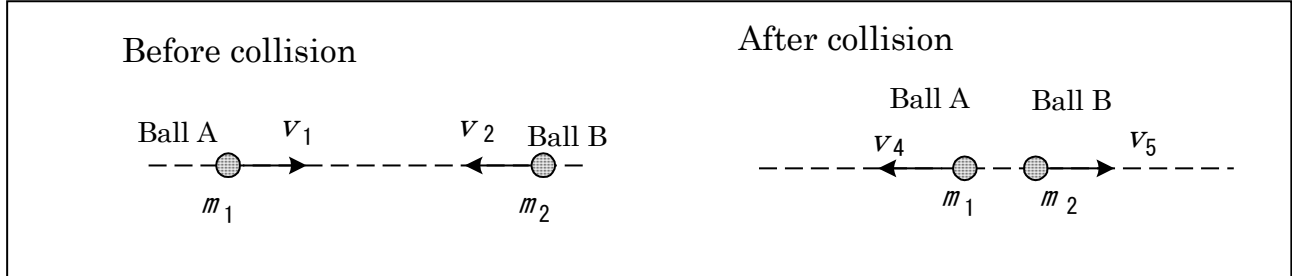
$$m_1 v_1 - m_1 v_3 = \int F dt \quad , \quad m_2 v_2 - m_2 v_3 = \int F' dt$$

From the law of action and reaction  $F = -F'$

$$\therefore m_1(v_1 - v_3) = m_2(v_3 - v_2) \quad \dots \quad (1)$$

$$\therefore m_1v_1 + m_2v_2 = (m_1 + m_2)v_3$$

$$v_3 = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad \dots \quad (2)$$



**Figure 4. To solve the collision of a perfect elastic ball using a linear equation for speed.** The mass of two elastic balls A and B are  $m_1$  and  $m_2$  respectively . Their speeds before collision are  $v_1$  and  $v_2$ . They collide on a straight line and after collision the speeds at which they fly away along the same straight line in the reverse direction are  $v_4$  and  $v_5$ , respectively.

Since they are perfect elastic balls, impulse  $\int Fdt$  is not lost and the same quantity of impulse in the reverse direction acts again on each ball. Each ball flies away with the speed  $v_4$  and  $v_5$ . Therefore we obtain for ball A.

$$m_1(v_1 - v_3) = m_1(v_3 - v_4)$$

$$\therefore v_4 = 2v_3 - v_1 \quad \dots \quad (3)$$

Substituting equation (2) for (3)

$$v_4 = 2 \frac{m_1v_1 + m_2v_2}{m_1 + m_2} - v_1 \quad \dots \quad (4)$$

similarly for ball B

$$m_2(v_3 - v_2) = m_2(v_5 - v_3)$$

$$\therefore v_5 = 2v_3 - v_2 \quad \dots \quad (5)$$

Substituting equation(2) for (5)

$$v_5 = 2 \frac{m_1v_1 + m_2v_2}{m_1 + m_2} - v_2 \quad \dots \quad (6)$$

Modifying equation(4), we therefore have

$$v_4 - \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = - \left\{ v_1 - \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \right\}$$

This equation means that speed  $v_1$  and  $v_4$  are same in quantity but opposite in direction from the viewpoint of a barycentric system. This solution employs neither the conventional law of conservation



of energy nor the law of conservation of momentum. This result must demonstrate a law of action and reaction, and conservation of impulse. That is to say, a solution is possible by means of a linear equation concerning speed.

Modifying equations (4) and (6)

$$v_4 = \frac{2m_2v_2}{m_1 + m_2} - \frac{(m_2 - m_1) \cdot v_1}{m_1 + m_2}$$

$$v_5 = \frac{2m_1v_1}{m_1 + m_2} - \frac{(m_1 - m_2) \cdot v_2}{m_1 + m_2}$$

we obtain the same result as by ordinary calculation methods.

### Explanation of the Compton Effect.

Since electromagnetic waves have momentum, the Compton Effect may be explained as an interaction with an electron in the above manner. In this case, the problem arises as to what is an appropriate view of a barycentric coordinate system of the interaction of an electron and light. It may be considered as a speed system in which the momentum of both are same in quantity and opposite in direction from the viewpoint of a barycentric coordinate system.

As shown in Figure 5, here  $\nu$  is the frequency of an incident ray,  $v$  the speed of the barycentric coordinate system composed of an incident wave and an electron,  $\nu'$  the frequency of an incident wave seen from the barycentric coordinate system,  $h$  is Planck's constant, and  $m$  is the mass of the electron. We obtain

$$mv = \frac{h\nu'}{c} \quad \dots (7)$$

In this case we may treat the frequency as a Doppler Effect

$$\nu' = \nu \left(1 - \frac{v}{c}\right) \quad \dots (8)$$

Substituting equation (8) for (7)

$$mv = \frac{h\nu}{c} \left(1 - \frac{v}{c}\right) \quad \dots (9)$$

Equation (9) is modified into

$$\therefore mv + \frac{h\nu}{c^2}v = \frac{h\nu}{c}$$

Then the speed of the barycentric coordinate system is

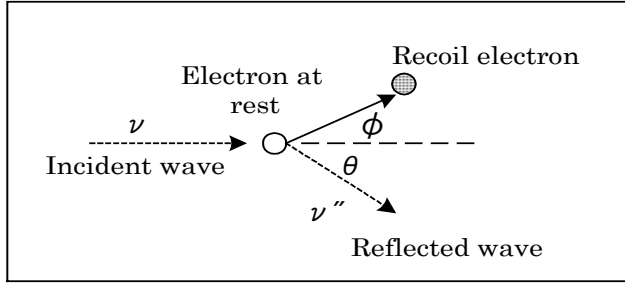
$$\therefore v = \frac{h\nu}{mc} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}} \quad \dots (10)$$

Substituting equation (10) for equation (8), the frequency of the incident wave seen from the barycentric coordinate system is described as

$$\nu' = \nu \left(1 - \frac{v}{c}\right) = \nu \left(1 - \frac{h\nu}{mc^2} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}}\right) = \frac{\nu}{1 + \frac{h\nu}{mc^2}} \quad \dots (11)$$

From this barycentric speed system we obtain the reflective wave with frequency  $\nu'$ . Therefore if it is observed from direction  $\theta$ , it may be considered as the Doppler Effect of  $\nu' \cos \theta$ . If  $\nu''$  is the frequency, we may state

$$\nu'' = \nu' \frac{c}{c - v \cos \theta} = \frac{\nu'}{1 - \frac{v}{c} \cos \theta} \quad (12)$$



**Figure 5 The interaction of an electron at rest and an electromagnetic wave.** A wave is reflected in the direction of  $\theta$  and a recoil electron moves in the direction of  $\phi$ . The frequency of the incident wave is  $\nu$ , and the frequency of the reflective wave is  $\nu''$ .  $h$  is Planck's constant,  $m$  is the mass of the electron, and  $c$  is the speed of light.  $\theta$  is the angle made by the reflective wave and the incident wave.

Substituting equations (10) and (11) for (12)

$$\nu'' = \frac{\nu}{1 + \frac{h\nu}{mc^2}} \cdot \frac{1}{1 - \frac{h\nu}{mc^2} \cos \theta} = \frac{\nu}{1 + \frac{h\nu}{mc^2} - \frac{h\nu}{mc^2} \cos \theta} \dots (13)$$

$$\therefore \nu'' + \frac{h\nu}{mc^2} (1 - \cos \theta) \nu'' = \nu$$

$$\therefore \nu - \nu'' = \frac{h\nu}{mc^2} \nu'' (1 - \cos \theta)$$

Dividing both sides by  $\nu \nu''$

$$\frac{1}{\nu''} - \frac{1}{\nu} = \frac{h}{mc^2} (1 - \cos \theta)$$

$$\frac{c}{\nu''} - \frac{c}{\nu} = \frac{h}{mc} (1 - \cos \theta) \dots (14)$$

With  $\lambda = c / \nu$  and  $\lambda'' = c / \nu''$  and substituting for equation(14)

$$\lambda'' - \lambda = \frac{h}{mc} (1 - \cos \theta)$$

This is the formula of the Compton Effect itself .

Three cases treated by means of the Lorentz transformation within the special theory of relativity have been explained without the use of this transformation. Therefore there is no necessity to consider concepts such as length contraction or expansion of time resulting from differing speed systems.

### The momentum and speed of a recoil electron in the Compton Effect

Above, the Compton Effect has been explained nonrelativistically. In this case, the frequency of a reflective wave decreases, therefore part of the energy of an incident wave is lost. We may expect that this result will lead to an equation showing the relation between the momentum and the velocity of the recoil electron. Next we attempt to determine the relation between the momentum of a recoil electron and speed using the results of the Compton Effect

### Examination of the speed of a barycentric coordinate system in the Compton Effect

The speed of a barycentric coordinate system consisting of an electron at rest and an incident wave may be represented following equation(10) as

$$v = \frac{1}{m} \cdot \frac{h\nu}{c} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}}$$

In this equation, the speed of a barycentric coordinate system is represented as the product of the ratio  $1/(1 + h\nu/mc^2)$  and the momentum of an incident wave  $h\nu/c$  and the reciprocal of the electron mass  $m$ . Accordingly,  $h\nu/mc^2$  in the denominator must express a ratio. Since the numerator expression  $h\nu$  indicates energy, then the denominator  $mc^2$  must also express energy. If expression  $h\nu/mc^2$  is modified into  $(h\nu/c)/mc$ , then the numerator expression  $h\nu/c$  indicates momentum, and therefore the denominator  $mc$  must also express momentum. In this case, since momentum is a matter of the difference of momentum, it is more appropriate to treat this as impulse. Since the momentum of an electromagnetic wave and the momentum of a particle are treated equally, an electron at rest must have energy  $mc^2$  or, alternatively, impulse  $mc$ .

### The momentum of a recoil electron in the Compton Effect

We may consider that the change in the momentum of an incident wave and a reflective wave is transferred to an electron according to the law of conservation of momentum. Here for convenience, we shall examine the case in which the direction of the reflective wave is opposite to that of an incident wave, that is the case where  $\theta = \pi$  in the Compton Effect.  $\nu$  is the frequency of the incident wave,  $\nu''$  the frequency of the reflective wave,  $v_c$  the speed of the barycentric coordinate system of an electron at rest and the incident wave, and  $v$  to the speed of a recoil electron. We find

$$v_c = \frac{h\nu}{mc} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}} \quad \dots \quad (15)$$

$$\nu'' = \frac{\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)} = \frac{\nu}{1 + 2\frac{h\nu}{mc^2}} \quad \dots \quad (16)$$

The quantity of the change of momentum of the electromagnetic wave is

$$\frac{h\nu}{c} + \frac{h\nu''}{c} = \frac{h\nu}{c} + \frac{h\nu}{c} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}} = \frac{h\nu}{c} \cdot \frac{2(1 + \frac{h\nu}{mc^2})}{1 + 2\frac{h\nu}{mc^2}} \quad \dots \quad (17)$$

On the other hand, the speed of an electron is considered to be twice the speed of the barycentric system, that is  $v_5$  in the case of equation (6) where  $v_2 = 0$ . According to equation (15) we find

$$mv = 2mv_c = \frac{h\nu}{c} \cdot \frac{2}{1 + \frac{h\nu}{mc^2}} \quad \dots \quad (18)$$

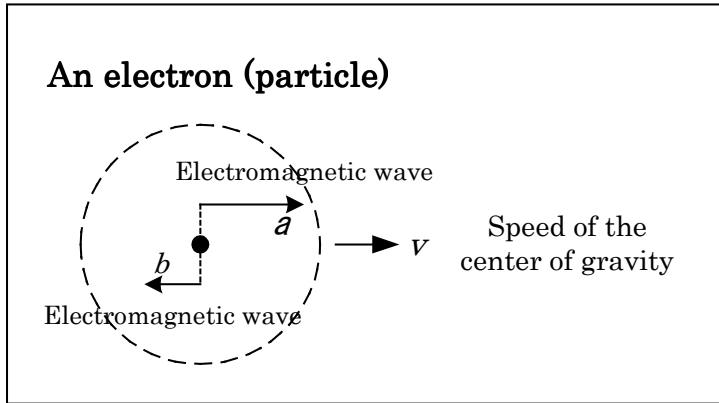
Equation (17) does not correspond to equation (18), so that the law of conservation of momentum will not be realized and the result will be contradictory. Equation (18) means that the momentum of the electron is exactly twice the momentum on the speed of the barycentric system, and it is for this reason that disagreement occurs.

### Reexamination of the concept of momentum of an electron

It appears that it is problematical to consider the momentum of an electron as the product of mass and speed. To solve this inconsistency we apply the contrary regarding and consider the momentum of an electromagnetic wave as  $h\nu/c$ , so that the momentum of an electron is connected to frequency and treated as wave motion. In which case, why are a state of rest and various speeds observed, instead of motion by the speed of light? The wave motion appears to be constrained in some way. Let

us consider an electron as the two electromagnetic waves moving in opposite directions which are joined by some cause and move while rotating around a centre of gravity as shown in Figure 6.

In this case, the centre of gravity of the two waves stands still, or takes on various speeds, and it appears that the speed of the centre of gravity of the two waves is determined by the Doppler Effect. Here  $v$  is the speed of the centre of gravity, and  $a$  and  $b$  the momentum of the waves.



**Figure 6. A new concept of the momentum of an electron.** Two electromagnetic waves moving in opposite directions join together, and move while rotating around a centre of gravity. This centre of gravity is observed as an electron (particle).  $v$  is the speed of the centre of gravity,  $a$  and  $b$  are momentum

We assume

$$a(1 - \frac{v}{c}) = b(1 + \frac{v}{c}) \quad \dots \quad (19) \quad (a > b)$$

The speed of the centre of gravity is shown to be the speed of the electron. If the electron is at rest in equation (19) then  $v=0$  so that  $a = b$ . Corresponding with the result of the section on “**Examination of the speed of a barycentric coordinate system in the Compton Effect**”  $a + b = mc$ , we find  $a = b = mc/2$ . Let us now attempt to determine the speed  $v_c$  of such a centre of gravity system. If an electromagnetic wave with frequency  $\nu$  acts on the left side of equation (19), then:

$$\begin{aligned} & (\frac{1}{2}mc + \frac{h\nu}{c})(1 - \frac{v_c}{c}) = \frac{1}{2}mc(1 + \frac{v_c}{c}) \\ \therefore \frac{1}{2}mc + \frac{h\nu}{c} - (\frac{1}{2}mc + \frac{h\nu}{c})\frac{v_c}{c} &= \frac{1}{2}mc + \frac{1}{2}mc \cdot \frac{v_c}{c} \\ \therefore \frac{h\nu}{c} &= (mc + \frac{h\nu}{c})\frac{v_c}{c} \\ \therefore v_c &= \frac{h\nu}{mc} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}} \quad \dots \quad (20) \end{aligned}$$

The resulting equation shows the speed of the barycentric coordinate system. Consequently it can be written as an equation of the speed of the electron after emitting a reflective wave:

$$(\frac{1}{2}mc + \frac{h\nu}{c})(1 - \frac{v}{c}) = (\frac{1}{2}mc - \frac{h\nu''}{c})(1 + \frac{v}{c}) \quad \dots \quad (21)$$

in which  $\nu''$  represent the frequency of the reflective wave.

### Determining the relation between the speed and momentum of an electron by the new concept

As shown in Figure 5, a wave is reflected in direction  $\theta$  and a recoil electron moves in direction  $\phi$ .  $\nu$

is the frequency of the incident wave,  $\nu''$  the frequency of the reflective wave.  $m$  is the mass of an electron at rest,  $c$  the speed of light,  $h$  Planck's constant, and  $a$  and  $b$  are impulses. We can now write the equation showing the speed of an electron as

$$a\left(1 - \frac{v}{c}\right) = b\left(1 + \frac{v}{c}\right) \quad \dots \quad (22)$$

Since the total impulse is conserved, we find

$$a + b = mc + \frac{h\nu}{c} - \frac{h\nu''}{c} \quad \dots \quad (23)$$

in which  $\nu$  represents the frequency of an incident wave and  $\nu''$  the frequency of a reflective wave. The impulse  $U$  received by an electron is

$$U = \frac{h\nu}{c} - \frac{h\nu''}{c} \quad \dots \quad (24)$$

Substituting (24) for (23), then

$$a + b = U + mc \quad \dots \quad (25)$$

Since  $(a - b)$  is the momentum received by the electron, by the law of conservation of momentum we can state

$$(a - b) \cos \phi = \frac{h\nu}{c} - \frac{h\nu''}{c} \cos \theta \quad \dots \quad (26)$$

$$(a - b) \sin \phi = \frac{h\nu''}{c} \sin \theta \quad \dots \quad (27)$$

Further, according to the Compton Effect

$$\frac{h\nu''}{c} = \frac{h\nu}{c} \cdot \frac{1}{1 + \frac{h\nu}{mc^2}(1 - \cos \theta)} \quad \dots \quad (28)$$

Processing  $(26)^2 + (27)^2$

$$\begin{aligned} (a - b)^2 \cos^2 \phi + (a - b)^2 \sin^2 \phi &= \left(\frac{h\nu}{c}\right)^2 - 2 \frac{h\nu}{c} \cdot \frac{h\nu''}{c} \cos \theta + \left(\frac{h\nu''}{c}\right)^2 \\ \therefore (a - b)^2 &= \left(\frac{h\nu}{c}\right)^2 - 2 \frac{h\nu}{c} \cdot \frac{h\nu''}{c} \cos \theta + \left(\frac{h\nu''}{c}\right)^2 \quad \dots \quad (29) \end{aligned}$$

Modifying equation (22) to

$$(a - b) = \frac{v}{c}(a + b)$$

and substituting equation (25) here, we obtain

$$\therefore (a - b) = \frac{v}{c}(U + mc) \quad \dots \quad (30)$$

Further substituting equation (30) for equation (29)

$$\therefore \left(\frac{v}{c}\right)^2 (U + mc)^2 = \left(\frac{h\nu}{c}\right)^2 - 2 \frac{h\nu}{c} \cdot \frac{h\nu''}{c} \cos \theta + \left(\frac{h\nu''}{c}\right)^2 \quad \dots \quad (31)$$

Equation (28) modifies into

$$\begin{aligned} \frac{h\nu''}{c} + \frac{h\nu}{mc^2} \cdot \frac{h\nu''}{c} (1 - \cos \theta) &= \frac{h\nu}{c} \\ \frac{h\nu}{mc^2} \frac{h\nu''}{c} - \frac{h\nu}{mc^2} \frac{h\nu''}{c} \cos \theta &= \frac{h\nu}{c} - \frac{h\nu''}{c} \end{aligned}$$

Multiplying both sides of the equation by  $(mc)$

$$\frac{h\nu}{c} \frac{h\nu''}{c} - \frac{h\nu}{c} \frac{h\nu''}{c} \cos\theta = mc\left(\frac{h\nu}{c} - \frac{h\nu''}{c}\right)$$

$$\therefore \frac{h\nu}{c} \frac{h\nu''}{c} \cos\theta = \frac{h\nu}{c} \frac{h\nu''}{c} - mc\left(\frac{h\nu}{c} - \frac{h\nu''}{c}\right) \quad \dots \quad (32)$$

Substituting equation (32) for (31) with rearrangement

$$\left(\frac{\nu}{c}\right)^2 (U + mc)^2 = \left(\frac{h\nu}{c} - \frac{h\nu''}{c}\right)^2 + 2mc\left(\frac{h\nu}{c} - \frac{h\nu''}{c}\right)$$

Substituting equation (24)

$$\therefore \left(\frac{\nu}{c}\right)^2 (U + mc) = U^2 + 2mcU$$

$$= (U + mc)^2 - (mc)^2 \quad \dots \quad (33)$$

$$\therefore (U + mc)^2 = \frac{mc^2}{1 - (\nu/c)^2} \quad \dots \quad (34)$$

Consequently

$$\therefore U + mc = \pm \frac{mc}{\sqrt{1 - (\nu/c)^2}} \quad \dots \quad (35)$$

Thus, without making use of the special theory of relativity, we can derive a relational expression between the speed and momentum of an electron from the result of the Compton Effect. This equation is similar to that for the increase of mass in the special theory of relativity.

Next we attempt to determine a relational expression between momentum  $a$  and momentum  $b$ . First, substituting equation (25) for (34) we obtain

$$(U + mc)^2 = (a + b)^2 = \frac{(mc)^2}{1 - \left(\frac{\nu}{c}\right)^2} \quad \dots \quad (36)$$

From equation (22)

$$\frac{\nu}{c} = \frac{a - b}{a + b} \quad \dots \quad (37)$$

Substituting this for equation (36), we can state

$$(a + b)^2 = \frac{(mc)^2}{1 - \left(\frac{a - b}{a + b}\right)^2}$$

$$\therefore (a + b)^2 - (a - b)^2 = (mc)^2 \quad \dots \quad (38)$$

$$\therefore 4ab = (mc)^2$$

$$\therefore ab = \left(\frac{1}{2}mc\right)^2 \quad \dots \quad (39)$$

This indicates conservation of the product  $(ab)$ . What kind of physical quantity corresponds to this conservation of product, we do not discuss here.

### Determining the relation between the speed and momentum of an electron in terms of energy

In the discussion in the section on “**Examination of the speed of a barycentric coordinate system in the Compton Effect**” impulse and energy were treated as equivalent quantities, and it was seen that the same results could be derived arguing in terms of energy instead of impulse. Thus the argument of

the preceding section can be recast in terms of energy.

The equation showing the speed of an electron is

$$a(1 - \frac{v}{c}) = b(1 + \frac{v}{c}) \quad \dots \quad (40) \quad : \text{ (hereafter, "the equation of speed")}$$

in which  $a$  and  $b$  represent energy. Since the total energy is conserved, we find

$$a + b = mc^2 + h\nu - h\nu'' \quad \dots \quad (41)$$

in which  $\nu$  represents the frequency of an incident wave and  $\nu''$  the frequency of a reflective wave.

The energy  $U$  received by an electron is

$$U = h\nu - h\nu'' \quad \dots \quad (42)$$

Substituting (42) for (41)

$$a + b = U + mc^2 \quad \dots \quad (43)$$

Since  $(a - b) / c$  is the momentum received by the electron, by the law of conservation of momentum we can state

$$\frac{(a - b)}{c} \cos \phi = \frac{h\phi}{c} - \frac{h\nu''}{c} \cos \theta \quad \dots \quad (44)$$

$$\frac{(a - b)}{c} \sin \phi = \frac{h\nu''}{c} \sin \theta \quad \dots \quad (45)$$

Further according to the Compton Effect

$$h\nu'' = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos \theta)} \quad \dots \quad (46)$$

Processing (44)<sup>2</sup> + (45)<sup>2</sup>

$$(a - b)^2 \cos^2 \phi + (a - b)^2 \sin^2 \phi = (h\nu)^2 - 2h\nu \cdot h\nu'' \cos \theta + (h\nu'')^2$$

$$\therefore (a - b)^2 = (h\nu)^2 - 2h\nu \cdot h\nu'' \cos \theta + (h\nu'')^2 \quad \dots \quad (47)$$

Modifying equation (40) to

$$(a - b) = \frac{v}{c}(a + b)$$

and substituting equation (43) here, we obtain

$$\therefore (a - b) = \frac{v}{c}(U + mc^2) \quad \dots \quad (48)$$

Further substituting equation (48) for (47)

$$\therefore \left(\frac{v}{c}\right)^2 (U + mc^2)^2 = (h\nu)^2 - 2h\nu \cdot h\nu'' \cos \theta + (h\nu'')^2 \quad \dots \quad (49)$$

Equation (46) modifies into

$$h\nu'' + \frac{h\nu}{mc^2} \cdot h\nu''(1 - \cos \theta) = h\nu$$

$$\therefore \frac{h\nu}{mc^2} h\nu'' - \frac{h\nu}{mc^2} h\nu'' \cos \theta = h\nu - h\nu''$$

Multiplying both the side of equation by  $(mc^2)$

$$\therefore h\nu \cdot h\nu'' - h\nu \cdot h\nu'' \cos \theta = mc^2(h\nu - h\nu'')$$

$$\therefore h\nu \cdot h\nu'' \cos \theta = h\nu \cdot h\nu'' - mc^2(h\nu - h\nu'') \quad \dots \quad (50)$$

Substituting equation (50) for (49) with re-arrangement

$$\begin{aligned}
 \left(\frac{v}{c}\right)^2 (U + mc^2)^2 &= (hv)^2 - 2hv \cdot hv'' + 2mc^2(hv - hv'') + (hv'')^2 \\
 &= (hv - hv'')^2 + 2mc^2(hv - hv'') \\
 &= U^2 + 2mc^2U \\
 &= (U + mc^2)^2 - (mc^2)^2 \quad \dots \quad (51)
 \end{aligned}$$

$$\therefore (U + mc^2)^2 = \frac{(mc^2)^2}{1 - (v/c)^2} \quad \dots \quad (52)$$

Consequently

$$U + mc^2 = \pm \frac{mc^2}{\sqrt{1 - (v/c)^2}} \quad \dots \quad (53)$$

This equation is exactly the same as the result of the special theory of relativity. If the concepts discussed here are applied to an electron, impulse and energy can be equally treated like an electromagnetic wave. Next we attempt to determine a relational expression between energy  $a$  and energy  $b$ . First, substituting equation (43) for (52) we obtain

$$(U + mc^2)^2 = (a + b)^2 = \frac{(mc^2)^2}{1 - \left(\frac{v}{c}\right)^2} \quad \dots \quad (54)$$

From equation (40)

$$\frac{v}{c} = \frac{a - b}{a + b} \quad \dots \quad (55)$$

Substituting this for equation (54), we can state

$$\begin{aligned}
 (a + b)^2 &= \frac{(mc^2)^2}{1 - \left(\frac{a - b}{a + b}\right)^2} \\
 (a + b)^2 \left\{ 1 - \left(\frac{a - b}{a + b}\right)^2 \right\} &= (mc^2)^2 \\
 \therefore (a + b)^2 - (a - b)^2 &= (mc^2)^2 \quad \dots \quad (56)
 \end{aligned}$$

$$\begin{aligned}
 \therefore 4ab &= (mc^2)^2 \\
 \therefore ab &= \left(\frac{mc^2}{2}\right)^2 \quad \dots \quad (57)
 \end{aligned}$$

This result also merely substitutes impulse ( $mc$ ) for energy ( $mc^2$ ).

## Conclusion

- Three cases treated by the Lorentz transformation within the special theory of relativity have been explained without the use of this transformation. Therefore there is no necessity to consider concepts such as length contraction or expansion of time resulting from differing speed systems.
- From the explanation of the Michelson-Moley experiment, because the negation of the existence of the ether as the medium of light or the electromagnetic wave is not possible, the ground of the special theory of relativity would be lost.
- The duality of particle substance and wave motion has so far been assumed in the world of physics.



If one assumes that one form of a wave is observed as a substance (particle), then a substance itself can be a wave motion.

- We have argued on the premise that the medium of an electromagnetic wave exists. (i.e. on the premise of the existence)
- Considering an electron (particle) to be one form of an electromagnetic wave, we have obtained an equation which indicates the speed by impulse in the Doppler Effect as wave motion.
- The equation showing the relation between the speed and impulse of an electron is

$$a\left(1 - \frac{v}{c}\right) = b\left(1 + \frac{v}{c}\right)$$

where  $(\mathbf{a} - b)$  indicates momentum and  $(\mathbf{a} + b = mc)$  indicates total impulse. Even in a state of rest, an electron has impulse  $(mc)$ , and the product indicated in the equation

$$ab = (mc / 2)^2$$

is conserved regardless of its speed. Moreover the total impulse also increases according to

$$U + mc = \frac{mc}{\sqrt{1 - (v/c)^2}}$$

We have obtained the same result as for the increase of mass in the special theory of relativity.

- The equation for wave motion speed can be expressed with energy in the same way as that for impulse, that is as

$$a\left(1 - \frac{v}{c}\right) = b\left(1 + \frac{v}{c}\right)$$

In this equation,  $(\mathbf{a} - b) / c$  indicates momentum, and  $(\mathbf{a} + b)$  indicates total energy. Even in a state of rest, an electron has energy  $(\mathbf{a} + b = mc^2)$ , and the product indicated by equation

$$ab = (mc^2 / 2)^2$$

is conserved regardless of its speed. Moreover the total energy also increases according to

$$U + mc = \frac{mc}{\sqrt{1 - (v/c)^2}}$$

We have again obtained the same result as for the increase of mass in the special theory of relativity. Therefore impulse and energy can be considered as the same physical quantity in the equation for wave motion speed

- The equation showing the speed of an electron ( expression of wave motion speed ) is

$$a\left(1 - \frac{v}{c}\right) = b\left(1 + \frac{v}{c}\right)$$

where the speed of an electron is determined by the quantity of the received impulse or energy. The speed of an electron does not change without a change in impulse or energy, so that it can be considered to manifest the law of inertia. The equation further indicates the resistance of an electron to change of speed, and may therefore be considered as relating to the origin of mass.